Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 12, 2003*
Revised: *March 19, 2004*
Accepted: *March 24, 2004*

Supersymmetry and the AdS Higgs phenomenon

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Abstract: We examine the AdS Higgs phenomenon for spin-1 fields, and demonstrate that graviphotons pick up a dynamically generated mass in AdS_4, once matter boundary conditions are relaxed. We perform an explicit one-loop calculation of the graviphoton mass, and compare this result with the mass generated for the graviton in AdS. In this manner, we obtain a condition for unbroken supersymmetry. With this condition, we examine both $\mathcal{N}=2$ and $\mathcal{N}=4$ gauged supergravities coupled to matter multiplets, and find that for both cases the ratio between dynamically generated graviton and graviphoton masses is consistent with unbroken supersymmetry.

Keywords: Spontaneous Symmetry Breaking, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

It has long been known that field theories in curved spacetime may exhibit interesting behavior without corresponding flat space analogs. While field theories in maximally symmetric spaces have already been well explored, recent developments in (A)dS/CFT and cosmology have led to renewed interest in field theories in de Sitter and anti-de Sitter spaces. For the latter, it is noteworthy that AdS space is not globally hyperbolic because of its timelike boundary, which is easily reached in finite time by null geodesics. Traditionally, the boundary of AdS has been dealt with by imposing reflecting boundary conditions, which is natural from the point of view of information flow [1]. However, investigations of the holographic dual to the Karch-Randall model [2] has led to the realization that the presence of multiple holographic domains [3, 4] naturally leads to transparent boundary conditions for the AdS field theory on the brane.

Thereafter, it was demonstrated by Porrati that a simple field theory in AdS with transparent boundary conditions coupled to Einstein gravity (with conventional reflecting boundary conditions for the graviton), leads to a dynamical generation of graviton mass [5]. This mechanism has been denoted the AdS Higgs phenomenon [5, 6], as the graviton gets mass by eating a composite Goldstone vector, which is a kinematically bound state of the field theory particles. While the one-loop computation of [5] was for a single conformally coupled scalar with generalized boundary conditions, it was subsequently demonstrated in [7] that matter fields of spins 0, 1/2 and 1 all contribute towards the graviton mass. In particular, viewing the holographic dual to the Karch-Randall model as a $\mathcal{N} = 4$ super-Yang-Mills CFT with $\mathrm{U}(N)$ gauge group, it was shown in [7] that the one-loop AdS Higgs computation correctly reproduces the result for the Karch-Randall graviton mass.

An important fact about the AdS Higgs phenomenon is that, while a mass is generated for the graviton, general covariance remains unbroken, and the gravitational Ward identities remain satisfied [5, 6]. Similarly, one may imagine from a supersymmetric context

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that if the graviton were to get a mass, its superpartners ought to become massive as well. In this sense, supersymmetry may remain unbroken, even with a dynamically generated gravitino mass, as in the case of general covariance and graviton mass. We will show that this is indeed what happens for supergravity coupled to a CFT, where the CFT fields are given unusual boundary conditions. This result is at least *consistent* with a supersymmetric realization of the Karch-Randall braneworld, with the entire localized supergravity multiplet originating from the quasi-zero-mode part of the Kaluza-Klein tower.

The AdS Higgs mechanism may be concisely stated in terms of SO(2,3) representation theory. For irreducible representations labeled by $D(E_0, s)$, massless representations in AdS_4 (as defined by propagation of a reduced number of states) generically corresponds to $D(E_0 = s+1, s)$, so that e.g. a massless graviton is given by $D(3,2)$ and a massless vector is given by $D(2,1)$. In this case, the AdS Higgs mechanism corresponds to the decomposition of a massive representation $D(E_0 > s+1, s)$ in the massless limit

$$\lim_{\epsilon \to 0} D(s+1+\epsilon, s) = D(s+1, s) + D(s+2, s-1) \qquad (s \geq 1)\,. \tag{1.1}$$

In particular, the graviton gets a mass by eating a massive $D(4,1)$ vector, while a graviphoton (or ordinary photon, for that matter) gets a mass by eating a minimally coupled $D(3,0)$ scalar. The latter case of course corresponds to the ordinary Higgs mechanism, whether in AdS or in flat space.

In this paper, we show that the graviphoton (in either $\mathcal{N} = 2$ or $\mathcal{N} = 4$ theories) indeed picks up a mass through the AdS Higgs phenomenon, and furthermore that its mass is related to the graviton mass in precisely the ratio demanded by the preservation of supersymmetry (for the massive spin-2 AdS multiplet). Of course, we expect the gravitino to become massive in just the same manner, although we have not performed an explicit check. This AdS Higgs phenomenon is in fact quite general, regardless of spin, and it seems fair to say that mass generation in AdS is a generic phenomenon any time boundary conditions are relaxed.

We begin in section 2 with a general discussion of the AdS Higgs phenomenon as it applies to mass generation for vector fields. The method for identifying the photon mass parallels that for the graviton mass worked out in [5, 7]. In section 3, we perform the actual one-loop calculation for both scalars and spin-1/2 fermions running through the loop. This is sufficient for examining graviphoton masses in $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supergravities, which we do in section 4. Finally, we conclude in section 5 by highlighting some of the main features of the AdS Higgs phenomenon.

2. The spin-1 AdS Higgs phenomenon

Although masslessness of the photon (or graviton) is traditionally associated with gauge invariance, the actual connection is rather more subtle. This was emphasized in [5] in the context of the AdS Higgs phenomenon. In fact, the kinematics of mass generation for a spin-1 vector is simpler than that of the graviton, and we review this here.

Ordinary field theory in AdS, or any curved space for that matter, is somewhat more involved than in a flat Minkowski background. In particular, lack of translational symmetry

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precludes a straightforward momentum space treatment. Nevertheless, we may proceed with a coordinate space analysis. If $\Sigma^\mu{}_\nu(x,y)$ denotes the vector boson self-energy, then gauge invariance ensures the transversality of $\Sigma^\mu{}_\nu$, so that it may be written in the form

$$\Sigma^\mu{}_\nu = \beta(\Delta)\Pi^\mu{}_\nu\,, \tag{2.1}$$

where $\beta(\Delta)$ is a scalar function (containing the dynamics, and undetermined by transversality) and $\Pi^\mu{}_\nu$ is the transverse projection

$$\Pi^\mu{}_\nu = g^\mu{}_\nu + \nabla^\mu \frac{1}{\Delta}\nabla_\nu\,. \tag{2.2}$$

Here, Δ denotes the Lichnerowicz operator which commutes with covariant differentiation, $[\Delta, \nabla_\mu]T_{\{\mu_i\}} = 0$. In particular, $\Delta^{(0)}\phi = -\nabla^2\phi$ and $\Delta^{(1)}A_\mu = -\nabla^2 A_\mu + R_{\mu\nu}A^\nu$ when acting on scalars and vectors, respectively.

Working in Landau gauge, the bare vector propagator takes the form

$$D^\mu{}_\nu = \frac{\Pi^\mu{}_\nu}{\Delta}\,. \tag{2.3}$$

Since $\Pi^\mu{}_\nu$ is a projection, the full propagator is easily resummed, yielding

$$\tilde{D}^\mu{}_\nu = \frac{\Pi^\mu{}_\nu}{\Delta - \beta(\Delta)}\,, \tag{2.4}$$

which is physical when evaluated between conserved currents. Any potential mass may then be read off by the shift in the pole of the propagator. Therefore it is the constant piece in the expansion $\beta(\Delta) = -M^2 + \mathcal{O}(\Delta)$ that yields a photon mass. This argument is simply the curved space analog of the standard textbook one, where $\Sigma^\mu{}_\nu = \beta(p^2)(\delta^\mu{}_\nu - p^\mu p_\nu/p^2)$ results in a resummed expression $\tilde{D} = \delta^\mu{}_\nu/(p^2+\beta(p^2))+\cdots$. Photon mass is then generated if $\beta(p^2) = -M^2 + \mathcal{O}(p^2)$. It is clear from the flat space context that a shift in the photon pole arises from a *non-local* self-energy expression. Likewise, this continues to be the case in curved space. This potential non-locality is the basis for identifying the actual value of the dynamically generated photon (or graviton) mass [5, 7].

While representation theory in AdS allows the possibility of an AdS Higgs phenomenon as indicated in (1.1), we must perform an actual loop computation to see that the generated mass is non-vanishing. This computation follows the techniques developed in [5, 7]. The method is essentially to compute the one-loop self energy, extract its non-local behavior, and then to identify the actual mass from the non-local data.

Let us actually consider the last point first, namely identifying the dynamically generated mass from the self-energy. We find it easiest to continue in homogeneous coordinates by embedding AdS_4 into R^5. In this case, AdS_4 is the restriction to the hyperboloid in R^5 given by $X^M X_M = -L^2$, where R^5 has metric $\eta_{MN} = \mathrm{diag}(-,+,+,+,-)$. We use X^M, Y^N $(M,N = 0,\ldots,4)$ to denote homogeneous coordinates and x^μ, y^ν $(\mu,\nu = 0,\ldots,3)$ to denote intrinsic coordinates on AdS_4. The metric on AdS_4 is then the projection of the R^5 metric onto the hyperboloid, $G^{MN}(X) = \eta^{MN} + X^M X^N/L^2$. Note that G^{MN} also serves to project vector quantities onto AdS_4.

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Scalar two-point functions $\phi(X,Y)$ in maximally symmetric spaces can only depend on the invariant interval between X and Y. This in turn may be expressed in terms of the R^5 distance $|X-Y|^2$. Since $X^2 = Y^2 = -L^2$ when restricted to AdS_4, this indicates that $\phi(X,Y)$ may be written in terms of a single scalar invariant $Z \equiv X \cdot Y/L^2$. Note that $|X-Y|^2/L^2 = -2(Z+1)$, so that the short distance region corresponds to taking $Z \to -1$. In fact, the hyperboloid in R^5 has two branches, and Z has the values $-\infty \le Z \le -1$ on the 'physical' AdS_4, while $1 \le Z \le \infty$ on the other branch. Recalling that AdS space is conformal to *half* of the Einstein static universe, the map $Z \to -Z$ (i.e. $Y \to -Y$) may then be viewed as a map taking Y to its image point on the Einstein static universe.

Vector functions in AdS_4 may be treated similarly using homogeneous coordinates. However, we must keep in mind to project all free indices onto AdS_4 using G^{MN}. As a result, symmetric vector two-point functions $\Sigma_{MN}(X,Y)$ may be written in terms of two invariant bi-vectors, $\hat{G}_{MN}(X,Y)$ and $N_M(X)N_N(Y)$, where

$$\begin{aligned}\hat{G}_{MN}(X,Y) &= G_{ML}(X)\eta^{LP}G_{PN}(Y) = \eta_{MN} + \frac{(X_M X_N + Y_M Y_N + Z X_M Y_N)}{L^2}\,,\\ N_M(X) &= \frac{Y_M + Z X_M}{L\sqrt{Z^2-1}}\,.\end{aligned} \tag{2.5}$$

These expressions are the direct generalizations of the flat space quantities $\eta_{\mu\nu}$ and $\hat{n}_\mu \hat{n}_\nu$ (where $\hat{n}$ is the unit vector pointing from x^μ to y^μ). For convenience, we may decompose $\Sigma_{MN}(X,Y)$ as

$$\Sigma_{MN} = a(Z)\left(\hat{G}_{MN} - Z N_M N_N\right) + b(Z) N_M N_N\,, \tag{2.6}$$

where it is to be understood that the index M refers to point X and the index N refers to point Y. In particular, arbitrary symmetric bi-vectors are specified by two independent scalar quantities $a(Z)$ and $b(Z)$.

Of course, transversality of the vector self-energy imposes a further condition on $a(Z)$ and $b(Z)$. To see this, we may take the covariant divergence $\nabla^M \Sigma_{MN} = G^{ML}\partial_M \Sigma_{LN}$ of (2.6) and demand that it vanishes. This yields a differential condition

$$a = \frac{1}{3}(Z^2-1)\frac{db(Z)}{dZ} + Z b(Z)\,, \tag{2.7}$$

so that the transverse self-energy may be completely specified by a single function $b(Z)$. We note that a large distance expansion of Σ_{MN} may be performed in terms of inverse powers of Z. In this case, it is convenient to introduce a set of asymptotic basis tensors T_i, so that $\Sigma^{MN} = \sum_i b_i T_i^{MN}$ where b_i are the constant coefficients $b(Z) = \sum_i b_i/Z^i$. The first few leading terms in the expansion are given in table 1.

	$a(Z)$	$b(Z)$
T_3	$0/Z^2 + 1/Z^4$	$1/Z^3$
T_4	$-1/3Z^3 + 4/3Z^5$	$1/Z^4$
T_5	$-2/3Z^4 + 5/3Z^6$	$1/Z^5$
T_6	$-1/Z^5 + 2/Z^7$	$1/Z^6$

Table 1: Leading basis tensors for the long distance expansion of the self-energy.

We now use these expressions to cast the transverse self-energy, (2.1), in a more useful manner. Focusing on the constant piece of $\beta(\Delta)$, we write

$$\Sigma_{\mu\nu} = -M^2 \Pi_{\mu\nu} = -M^2\left(g_{\mu\nu} + \nabla_\mu \frac{1}{\Delta}\nabla_\nu\right), \tag{2.8}$$

and assume that it acts between vector quantities, $\phi^\mu(x)\Sigma_{\mu\nu}(x,y)\phi^\nu(y)$. Since this would be integrated over all of space in order to compute physical quantities, we may integrate by parts to obtain

$$\begin{aligned}\int d^4x\, d^4y\, \phi^\mu(x)\Sigma_{\mu\nu}(x,y)\phi^\nu(y) &= -M^2 \int d^4x\, d^4y\, \phi^\mu(x) \times \\ &\qquad \times \left(g_{\mu\nu}\delta^4(x-y) - (\nabla_{x^\mu}\nabla_{y^\nu}\Delta^{-1}(x,y))\right)\phi^\nu(y)\,, \end{aligned} \tag{2.9}$$

where Δ^{-1} is the $E_0 = 3$ (minimally coupled) scalar Greens function in an AdS background.

Working in homogeneous coordinates, where the scalar Greens function has the form

$$\Delta^{-1}(Z) = \frac{1}{4\pi^2L^2}\left(\frac{Z}{Z^2-1} + \frac{1}{2}\log\frac{Z+1}{Z-1}\right) \qquad (E_0 = 3)\,, \tag{2.10}$$

and dropping the contact term, we find that the non-local part of Σ_{MN} has the form

$$\Sigma_{MN} = -M^2\nabla_M\nabla_N\Delta^{-1}(Z) = -\frac{M^2}{2\pi^2L^4(Z^2-1)^2}\left(\hat{G}_{MN} - 4ZN_MN_N\right). \tag{2.11}$$

This may be expanded in terms of the asymptotic bi-vectors given in table 1. The result is

$$\Sigma_{MN} = -\frac{M^2}{2\pi^2L^4}\left(3T_3 + 6T_5 + 9T_7\ldots\right). \tag{2.12}$$

Note that only odd T_i's show up in this expression. Finally, we see that the mass may be extracted from the leading asymptotic behavior of the self-energy according to $\Sigma_{MN} \sim (-3M^2/2\pi^2L^4)T_3 + \cdots$. In the next section we will compute scalar and spin-1/2 loop contributions to Σ, and in this manner find explicit expressions for the induced vector mass.

3. A one-loop computation of the graviphoton mass

For a non-abelian vector with gauge group G coupled to a conserved current $J^a_M(X)$, the self-energy is given by the two-point function $\Sigma^{ab}_{MN}(X,Y) = \langle J^a_M(X)J^b_N(Y)\rangle$. Here a and b denote gauge indices taking values in the adjoint of G. We take the currents to have the forms

$$\begin{aligned} J^a_{(0)\,M} &= \frac{ig}{2}\phi_i\left(\overrightarrow{\nabla}_M - \overleftarrow{\nabla}_M\right)(T^a)_{ij}\,\phi_j\,, \\ J^a_{(\frac{1}{2})\,M} &= ig\overline{\psi_i}\Gamma_M\,(S^a)_{ij}\,\psi_j\,, \end{aligned} \tag{3.1}$$

for couplings to real spin-0 and Dirac spin-1/2 fields, respectively. We take T^a and S^a to generate arbitrary (potentially reducible) representations of G, except that T^a must generate only real representations.

Evaluating the self-energy follows from Wick's theorem. In order to proceed, we need explicit forms for the scalar and spinor propagators in AdS. Since in the end we are interested in conformal matter, we consider only conformally coupled scalars and massless Dirac fields. Their respective propagators are given by [1]

$$\Delta_{(0)} = \frac{1}{8\pi^2 L^2}\left(\frac{\alpha_+}{Z+1} + \frac{\alpha_-}{Z-1}\right), \tag{3.2}$$

$$\Delta_{(\frac{1}{2})} = \frac{1}{8\pi^2 L^4}\left(\frac{\alpha_+ \Gamma^M (X_M - Y_M)}{(Z+1)^2} + \frac{\alpha_- \Gamma^M (X_M + Y_M)}{(Z-1)^2}\right), \tag{3.3}$$

where α_+ and α_- specify boundary conditions on AdS_4 [5, 7]. At short distances compared to the AdS radius L, the behavior of these propagators must match onto the corresponding flat-space ones. Since short distances correspond to $Z \to -1$, and the properly normalized four-dimensional behavior must be of the form

$$\Delta(Z \to -1) \sim -\frac{1}{4\pi^2 |X-Y|^2} = \frac{1}{8\pi^2 L^2}\frac{1}{Z+1}, \tag{3.4}$$

we see that this demands $\alpha_+ = 1$. On the other hand, α_- is undetermined at short distances. The expressions (3.2) and (3.3) clearly indicate an image charge structure on the Einstein static universe, with $\alpha_- = \pm 1$ corresponding to reflecting boundary conditions and $\alpha = 0$ to transparent ones.

We again concern ourselves only with the non local part, and so we only take Wick contractions between fields at different points. We take the propagators to be diagonal in representation space, so that Wick contractions give δ_{ij} in addition to the propagators of (3.2) and (3.3). For the spin-0 contribution to the self-energy, we find

$$\begin{aligned}\hat{\Sigma}^{ab}_{(0)\ MN} &= -\frac{1}{4} g^2 \left\langle \phi_i \nabla_M \phi_j \phi_k \nabla_N \phi_l \right\rangle \left((T^a)_{ij} - (T^a)_{ji}\right)\left((T^b)_{kl} - (T^b)_{lk}\right) \\ &= g^2 \, \mathrm{Tr}(T^a T^b) \left(-\Delta_{(0)} \nabla_M \nabla_N \Delta_{(0)} + \nabla_M \Delta_{(0)} \nabla_N \Delta_{(0)}\right). \end{aligned} \tag{3.5}$$

Working out the derivatives of the scalar propagator, (3.2), and dropping contact terms (since we are only interested in the long distance behavior), we find

$$\begin{aligned}\hat{\Sigma}^{ab}_{(0)\ MN} = & \frac{-g^2\,\mathrm{Tr}(T^a T^b)}{64\pi^4 L^6}\left[\frac{\alpha_+^2}{(Z+1)^3}\left(\left(\hat{G}_{MN} - Z N_M N_N\right) + N_M N_N\right) + \right. \\ & \quad + \frac{\alpha_-^2}{(Z-1)^3}\left(\left(\hat{G}_{MN} - Z N_M N_N\right) - N_M N_N\right) + \\ & \quad \left. + \frac{2\alpha_+ \alpha_- Z}{(Z^2-1)^2}\left(\left(\hat{G}_{MN} - Z N_M N_N\right) - 3 N_M N_N\right)\right] \\ = & \frac{-g^2\,\mathrm{Tr}(T^a T^b)}{64\pi^4 L^6}\left[\left((\alpha_+^2 - \alpha_-^2) T_3 - 3(\alpha_+^2 + \alpha_-^2) T_4 \ldots\right) + 2\alpha_+ \alpha_- \left(-3 T_4 \ldots\right)\right]. \end{aligned} \tag{3.6}$$

Since the induced photon mass is proportional to the coefficient of T_3, this demonstrates that it must vanish when reflecting boundary conditions are imposed on the scalars, $\alpha_+^2 = \alpha_-^2$. A mass is generated for all other cases (which incidentally correspond to non-unitary behavior at the field theory level, since in such cases not all of the information is reflected back from the AdS_4 boundary).

A similar computation for the fermion-loop contribution yields

$$\begin{aligned}\hat{\Sigma}^{ab}_{(\frac{1}{2})\,MN} &= -g^2\,\mathrm{Tr}(S^aS^b)\,\mathrm{Tr}\left(\Gamma_M\Delta_{(\frac{1}{2})}(X,Y)\Gamma_N\Delta_{(\frac{1}{2})}(Y,X)\right)\\ &= \frac{-g^2\,\mathrm{Tr}(S^aS^b)}{8\pi^4L^6}\left[\frac{\alpha_+^2}{(Z+1)^3}\left(\left(\hat{G}_{MN}-ZN_MN_N\right)+N_MN_N\right)+\right.\\ &\qquad\left.+\frac{\alpha_-^2}{(Z-1)^3}\left(\left(\hat{G}_{MN}-ZN_MN_N\right)-N_MN_N\right)\right]\\ &= \frac{-g^2\,\mathrm{Tr}(S^aS^b)}{8\pi^4L^6}\left[(\alpha_+^2-\alpha_-^2)T_3-3(\alpha_+^2+\alpha_-^2)T_4\ldots\right]. \end{aligned} \tag{3.7}$$

This expression is similar to that for the scalar loop, except for the absence of a mixed $\alpha_+\alpha_-$ term. This appears to be a result of working with Dirac spinors, and may not hold for Majorana ones. Nevertheless, this potential mixed term is irrelevant as far as mass generation is concerned, since it is of higher order in the asymptotic expansion.

As a result, we may obtain a universal expression for the induced graviphoton mass based on its coupling to conformal spin-0 and spin-1/2 fields. In order to preserve supersymmetry, the boundary conditions of all fields in a single supermultiplet must be chosen identically [8]. This allows us to combine the results of (3.6) and (3.7). Comparing with the leading behavior of (2.12), we therefore find that the dynamically generated mass of the graviphoton is given by

$$(M_1)^2 = g^2\frac{(\alpha_+^2-\alpha_-^2)}{96\pi^2L^2}\left(\mathrm{Tr}(T^aT^b)+4\,\mathrm{Tr}(S^aS^b)\right), \tag{3.8}$$

where the traces are now taken over real scalars (representation T^a) and Majorana fermions (representation S^a), respectively.

4. A check of supersymmetry

In general, unitary representations of AdS_4, given by $D(E_0,s)$, must satisfy a bound $E_0 \geq s+1$ for $s \geq 1$. Saturation of this bound corresponds to shortened ('massless') representations. As indicated in (1.1), the AdS Higgs mechanism is related to the fact that a massive representation becomes reducible in the limit $E_0 \to s+1$.

There is of course a supersymmetric generalization of the AdS Higgs phenomenon, where a complete supermultiplet may pick up a mass without explicitly breaking supersymmetry. Note that supersymmetry relates E_0 and s by units of 1/2 within a supermultiplet. In particular, the massless graviton and graviphotons are given by $D(3,2)$ and $D(2,1)$, respectively. While the massive representations of supersymmetry tend to be rather large, we will not need any explicit tabulations. We simply note that, for the supergravity multiplet including graviphotons, the AdS Higgs phenomenon yields the partial decomposition

$$D(3+\epsilon,2)+D(2+\epsilon,1)+\cdots = (D(3,2)+D(4,1))+(D(2,1)+D(3,0))+\cdots, \tag{4.1}$$

in the limit that $\epsilon\to 0$. The ellipses on both sides denote other members of the supermultiplet that we are not explicitly interested in. Note, however, that the mass shift (or rather E_0 shift) is given by the *same* parameter ϵ for all members of the multiplet.

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In this manner, we may check whether the graviphoton mass computed here agrees with the graviton mass computed previously in [5, 7]. Using the relations between mass and E_0 [9]

$$E_0^{(s=1)} = \frac{3}{2} + \frac{1}{2}\sqrt{1+(M_1 L)^2}\,, \qquad E_0^{(s=2)} = \frac{3}{2} + \frac{1}{2}\sqrt{9+(M_2 L)^2}\,, \tag{4.2}$$

and expanding for small masses, we find

$$\epsilon^{(s=1)} = \frac{1}{4}(M_1 L)^2\,, \qquad \epsilon^{(s=2)} = \frac{1}{12}(M_2 L)^2\,. \tag{4.3}$$

If ϵ were universal, as demanded by supersymmetry, this would indicate that $(M_2)^2 = 3(M_1)^2$. We now recall the result of [7], which gives

$$(M_2)^2 = 8\pi G_4 \frac{(\alpha_+^2 - \alpha_-^2)}{160\pi^2 L^4}(n_0 + 3n_{\frac{1}{2}} + 12 n_1)\,, \tag{4.4}$$

for the mass of the graviton. Here G_4 is the four-dimensional Newton's constant, and n_0, $n_{\frac{1}{2}}$ and n_1 are the total number of real spin-0, Majorana spin-1/2 and spin-1 fields. It is interesting to note that the cross terms proportional to $\alpha_+\alpha_-$ vanish in the graviton mass calculation because there is a relative sign difference in the α_- boundary condition between scalars and pseudoscalars, and they come in equal numbers in a supermultiplet. On the other hand, while such cross terms are present in the graviphoton calculation, they simply do not contribute to the mass term.

Finally, we combine (3.8) with (4.4) and the supersymmetry condition $(M_2)^2 = 3(M_1)^2$ to obtain

$$\frac{8\pi G_4}{5L^2}(n_0 + 3n_{\frac{1}{2}} + 12n_1) = g^2(S_2(T_0) + 4S_2(S_{\frac{1}{2}}))\,, \tag{4.5}$$

which is the resulting condition for unbroken supersymmetry. We have defined $\mathrm{Tr}(R^a R^b) = S_2(R)\delta^{ab}$, which is essentially the sum of the indices of the irreducible representations comprising R. While the number of fields, n_0 and $n_{\frac{1}{2}}$, do not show up explicitly on the right hand side, this information is contained in the fact that the scalar and spinor representations may be reducible. In particular, n_0 and $n_{\frac{1}{2}}$ simply counts the dimensions of the representations, so that $n_0 = \dim(T_0)$ and $n_{\frac{1}{2}} = \dim(S_{\frac{1}{2}})$. We have not considered vectors in the loop, as consistency of the non-abelian theory demands a uniform treatment of such vectors, and we do not modify the reflecting boundary conditions of the graviphotons themselves (so that there is no spin-1 contribution to the mass on the right hand side).

We now look at some specific examples of gauged supergravities admitting an AdS$_4$ vacuum solution. For matter coupled supergravities, there is often a choice of how much gauging may be turned on. Perhaps the simplest cases involve gauging only the R symmetry. However, even here one may not have enough graviphotons to gauge the entire group, so that only a subgroup of the full R symmetry may be gauged. After gauging, the graviphotons transform under the adjoint of the gauged R symmetry group, and couple to the corresponding R symmetry currents of the matter sector. In addition, turning on the gauging leads to a potential; for the theories of interest, the potential admits a stable AdS$_4$ vacuum. Since the strength of the potential is related to the gauge coupling, this allows us to rewrite g^2 in terms of the AdS$_4$ radius L and the four-dimensional Newton's constant G_4. Noting that $g^2 \sim G_4/L^2$, this allows a direct comparison of both sides of eq. (4.5).

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In the context of large-N gauge theories coupled to supergravity, it is only the matter fields that are expected to receive unusual (i.e. transparent) boundary conditions. As a result, only matter in the loop would give rise to dynamically generated masses, and we may ignore gravity multiplet self-contributions to its mass. In any case, potential graviton loop effects would be suppressed by $1/N^2$, and at least formally may be ignored.

The simplest example of AdS_4 gauged supergravity is the $\mathcal{N}=2$ model, where the $O(2)$ symmetry may be gauged by the graviphoton of the $\mathcal{N}=2$ gravity multiplet [10, 11]. Following the convention of [12], which couples this model to an arbitrary number of vector multiplets, we see that gauging yields a constant negative potential $-3g^2/2K^4$ where $4K^2 = 16\pi G_4$. Relating this potential to the cosmological constant, we find the gauge coupling constant to be given by $g^2 = 4\pi G_4/L^2$, so that the condition (4.5) becomes $\frac{2}{5}(n_0 + 3n_{\frac{1}{2}} + 12n_1) = (S_2(T_0) + 4S_2(S_{\frac{1}{2}}))$. The $\mathcal{N}=2$ vector multiplet is composed of one vector field, 2 Majorana spinors, and 2 scalars (one is a pseudoscalar). However only the Majorana spinors are charged under $O(2)$, transforming with generators $i\epsilon^{lm}$. This results in $S_2(S_{\frac{1}{2}}) = 2$ (as we should expect, since this Majorana doublet under $O(2)$ corresponds to one Dirac spinor under U(1)). With this counting, we find that the supersymmetry condition is met. Note, however, that while all members of the vector multiplet contribute to the graviton mass, only the gauginos contribute to the graviphoton mass.

Turning next to $\mathcal{N}=4$ gauged supergravity, we recall that there are two versions of the ungauged theory, with global SO(4) and SU(4) symmetries, respectively. Gauging of the latter theory yields the SU(2) $\times$ SU(2) Freedman-Schwarz model [13] with no stable extrema. On the other hand, gauging the SO(4) $\simeq$ SU(2) $\times$ SU(2) model using all six graviphotons yields a potential admitting an AdS_4 solution [14, 15]. It was further shown in [16] that different gauge couplings in each of the SU(2)'s may be obtained by field redefinitions of the theory where the gauge couplings were initially taken to be the same. Therefore, without loss of generality, we set the two SU(2) couplings to be equal.

For the standard $\mathcal{N}=4$ gauged model coupled to vector multiplets, the resulting potential receives contributions from the scalars in the vector multiplet. However, the AdS_4 extremum is given by the constant part of the potential, $V_0 = -12g^2/(16\pi G_4)$. Rewriting this in terms of the AdS_4 radius yields $g^2 = 16\pi G_4/2L^2$, and a resulting supersymmetry condition $\frac{1}{5}(n_0 + 3n_{\frac{1}{2}} + 12n_1) = (S_2(T_0) + 4S_2(S_{\frac{1}{2}}))$. Before proceeding, we must be careful to state how each field transforms under the SU(2) $\times$ SU(2) gauge group. The spinors transform as a $(2,2)$ and the scalars as a $(1,3)+(3,1)$. We use $\frac{i}{2}\vec{\sigma}$ and $i\epsilon^{ijk}$ as the generators of SU(2) working on spinors and scalars respectively. This gives that $S_2(T_0) = 2$ and $S_2(S_{\frac{1}{2}}) = 1$ (for either one of the two SU(2) factors). The check of supersymmetry then reads

$$\frac{1}{5}(6 + 3\times 4 + 12\times 1) = (2 + 4\times 1)\,, \tag{4.6}$$

which is indeed satisfied. As in the $\mathcal{N}=2$ case above, the contributions to the graviton and graviphoton masses arise in different combinations. However, the factors for the complete vector multiplet conspire to ensure supersymmetric mass generation for the gravity multiplet.

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5. Discussion

Although much of the investigation on the AdS Higgs phenomenon has been motivated by holography of the Karch-Randall model, it is worth emphasizing that the phenomenon does not depend on extra dimensions or other novel ideas, and is simply a result of looking at field theory in curved spacetimes. The reason the Higgs phenomenon has gone unnoticed for a long time is because it only shows up in the presence of unusual (and typically non-unitary) boundary conditions on the fields in AdS_4, despite the fact that transparent boundary conditions were known since at least the work of [1].

Of course, non-unitary boundary conditions are often dismissed as unphysical. However, another interpretation may be given: that the theory in AdS_4 by itself is incomplete, and that one must also include a defect field theory on the boundary of the AdS_4 spacetime [17, 18]. This point of view is natural in terms of holography with multiple domains, such as what happens in the Karch-Randall model [3, 4]. In this context, the condition for unbroken supersymmetry that we have derived, (4.5), provides an important check when considering the viability of a supersymmetric theory living on the brane. In particular, it suggests that the construction of an $\mathcal{N} = 4$ theory living on the Karch-Randall brane is indeed possible.

This dynamically generated mass for the supergravity multiplet scales as G_4/L^4, and vanishes in the flat space limit of AdS_4. This is of course to be expected, because in this limit, the focusing effect of anti-de Sitter space is lost, and the composite Goldstone boson responsible for the Higgs mechanism is no longer kinematically bound together. At the same time, boundary conditions at infinity lose their importance when AdS_4 is reduced to flat space.

We note that, while we have not directly computed the one-loop gravitino mass, such a calculation would be straightforward and may be related to the two-point function of the supercurrent, $\langle \mathcal{J}_\mu(x)\mathcal{J}_\nu(y)\rangle$. More explicitly, one may apply the same procedure of first identifying the appropriate transverse projections for the spin-3/2 field and then determining the general form of the two-point function compatible with the gauge symmetry. We expect the two-point function to have the form

$$\Sigma^\mu{}_\nu = \beta(\Delta)\Pi^\mu{}_\nu + \gamma(\Delta)K^\mu{}_\nu\,, \tag{5.1}$$

where $\Pi^\mu{}_\nu$ would be transverse and gamma-traceless while $K^\mu{}_\nu$ would be transverse but with gamma-trace. There is a slight subtlety in defining gravitino mass, however. Taking the spin-3/2 equation to be $(\gamma^{\mu\nu\rho}\nabla_\nu + M_{3/2}\gamma^{\mu\rho})\psi_\rho = 0$, the 'massless' gravitino in AdS_4 in fact has $M_{3/2} = 1/L$ (which is required for propagation with reduced degrees of freedom). This is also the required value of $M_{3/2}$ for consistent coupling to the supercovariantly conserved supercurrent, $\mathcal{D}^\mu\mathcal{J}_\mu = 0$, where $\mathcal{D}_\mu = \nabla_\mu + \frac{1}{2L}\gamma_\mu$.

A resummation of the gravitino propagator including the self energy (5.1) gives a shift in the pole of the full propagator whose interpretation would be a mass shift for the gravitino related to the functional form of $\beta(\Delta)$. Of course an actual one-loop computation of the two-point function $\langle \mathcal{J}_\mu(x)\mathcal{J}_\nu(y)\rangle$ would have to be performed, with mass extracted from its long distance behavior. This may be accomplished using the homogeneous coordinate techniques developed in refs. [5, 7] and the above sections of the present paper.

Since the supercurrent is in the same multiplet as the stress tensor and R-current, we expect the resulting gravitino mass to respect a similar supersymmetry condition as the one discussed in the previous section. Note, however, that the spin-3/2 mass and E_0 relation

$$E_0^{(s=3/2)} = \frac{3}{2} + |M_{3/2}L| \tag{5.2}$$

yields a linear shift in E_0 given by $\epsilon^{(s=3/2)} = M_{3/2}L - 1$ where $M_{3/2}L = 1$ is the 'massless' value mentioned above. This is in contrast with the quadratic shifts of (4.3). Including the gravitino mass, the supersymmetry preserving condition is then

$$(M_2L)^2 = 12(M_{3/2}L - 1) = 3(M_1L)^2 \,. \tag{5.3}$$

Although it may be somewhat surprising to consider unbroken supersymmetry with massive gravitinos, we emphasize that this is no more so than unbroken general covariance with a massive graviton or unbroken gauge invariance with massive photons.

Finally, we emphasize that the AdS Higgs phenomenon consistently generates masses for non-abelian gauge fields, and not just for abelian photons. This is clear from the example of gauged $\mathcal{N} = 4$ supergravity, where the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ graviphotons become massive. Although this Higgs phenomenon is purely an anti-de Sitter effect, related to unusual boundary conditions, it nevertheless provides a new gauge invariant mechanism for mass generation for non-abelian gauge bosons. Of course, such masses are generally small, on the order of the natural AdS scale. However, they may be controlled by making adjustments to the matter fields and their boundary conditions. While an actual AdS spacetime is disfavored by observation, it would still be curious to see if this Higgs phenomenon has any relevance to mass generation in the Standard Model.

Acknowledgments

This work was supported in part by the US Department of Energy under grant DE-FG02-95ER40899. JTL wishes to thank M. Duff, M. Porrati and H. Sati for discussions and D. Gross for raising the issue of mass generation for photons in AdS and for highlighting the kinematically bound state nature of the Goldstone boson.

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